A Model of Fed Information Effect

Hengjie Ai, and Xinxin Hu*

November 18, 2025

Abstract: A significant fraction of measured surprise central bank interest rate hikes is associated with simultaneous stock market run-ups and upward revisions in economic growth forecasts. This evidence is often interpreted as contradicting the standard New Keynesian transmission mechanism and as indicating that the Fed possesses superior information about economic fundamentals. We present a New Keynesian model in which investors are uncertain about the Fed's long-run monetary policy objectives and learn from observed Fed actions. Our model does not assume that the Fed has superior information, but it nonetheless generates a "Fed information effect," that is, a positive co-movement between interest rate surprises, revisions in expected growth, and stock returns as an equilibrium outcome. We show that the key predictions of our model are consistent with empirical evidence on asset market responses to measured Fed information shocks.

Keywords: Fed information, New Keynesian model, monetary policy expectation.

*Hengjie Ai (hengjie.ai@wisc.edu) is from Wisconsin School of Business, University of Wisconsin-Madison. Xinxin Hu (xhu344@wisc.edu) is from Wisconsin School of Business, University of Wisconsin-Madison.

1 Introduction

Recent literature on high-frequency identification of monetary policy often finds the sign of the impact of monetary policy surprises, as measured by surprises interest rate reductions from financial markets, on economic growth forecasts and stock market valuation to be time-varying (for example, Nakamura and Steinsson (2018), Bauer and Swanson (2023a)). This leads Nakamura and Steinsson (2018) to conclude the "Fed information effect". That is, the monetary authority has superior information about the state of the macroeconomy. As a result, monetary policy announcements not only reveal the Fed's plan about the future path of the macroeconomy, but also its superior information about macroeconomic fundamentals. However, evidence for a substantial information advantage of the Federal Reserve about the real economy over the private sector is rare.[1] If the Fed has superior information, it is more likely to be superior information about its own actions, rather than that about economic fundamentals.

In this paper, we present a model of the Fed information effect based on the Fed's private information about its own type. Our model is set in a standard New Keynesian framework, where monetary policy follows a Taylor rule with time-varying long-run interest rate targets. In this setup, the Fed's interest rate decisions depend on both its long-run targets and its short-run policy preferences. Although an inflation hawk targets a higher long-run interest rate, its short-run decisions may temporarily deviate. The market observes the Fed's policy decisions and infers its type when forming forward-looking pricing decisions.

As in standard New Keynesian models, nominal interest rate reductions lead to real interest rate reductions due to price stickiness. In equilibrium, real rate reductions must be associated with lower expected consumption growth. When uncertainty about the Fed's type is low, monetary policy affects the real economy in the same way as in standard New Keynesian models: temporary policy shocks do not affect the economy's steady state. As a result, the decline in expected consumption growth induced by a rate cut must be offset by a contemporaneous rise in consumption and output, along with a simultaneous increase in stock market valuation.

[1] For example, the Federal Reserve's Greenbook forecasts do not systematically outperform professional survey forecasts from Blue Chip. In addition, media discussions on FOMC decisions often emphasize interest rates decisions and not the information about macroeconomic variables revealed in interest rate decisions. See Bauer and Swanson (2023a).

When uncertainty about the Fed's type is high, a nominal interest rate reduction may arise as the result of a contractionary monetary policy shock and be associated with a stock market decline. When investors prior belief about Fed's long-run policy objective is imprecise, a contractionary monetary policy shock is interpreted as a signal for a tightening monetary policy regime in the long run due to Bayesian updating. The equilibrium nominal rate drops for two reasons. First, investors anticipate a long-run tighten monetary policy regime and as a result, a lower steady state level of consumption. As a result, the real interest rate drop due to a negative shock to expected consumption growth. This channel is absent from standard New Keynesian models because monetary policy does not affect the long-run steady state of the economy. In addition, tightening monetary policy shock lowers inflation and inflation expectations. The two effects work together to lower equilibrium nominal interest rate.

At the same time, this contractionary monetary shock lowers consumption, real wage and stock market valuation. Because stock market valuation is forward looking, a long run tighten monetary policy regime is associated with a lower steady-state level of output and hence a lower present value of dividends. In this scenario, nominal interest rate changes and stock market valuations move in the same direction and would be identified as "Fed information shocks" by the methodology of Jarocinski and Karadi (2020).

The key for our model to generate the "Fed information effect" is that belief updates of Fed type alter the relationship between interest rate and current-period output. In standard New Keynesian models, the effect of monetary policy shocks are temporary and they do not affect the long-run steady state of the economy. Intertemporal optimality requires that a lower real rate must be accompanied by lower expected consumption growth. The only way this can happen in equilibrium is that current period consumption rises, because monetary policy does not affect future consumption steady state. When Fed's type is a persistent state variable, however, monetary policy may affect the long-run steady state of the economy by impacting investor's belief about Fed's type. In the above example, a higher interest rate can be consistent with a higher current period consumption if investors belief that economy is transitioning to a higher long-run consumption steady state. Observationally, this is consistent with a Fed information effect because a higher interest rate is associated with upward revisions of future consumption and output.

The above mechanism for Fed information effect implies that Fed information shocks is more likely to happen when uncertainty about the long-run objectives of monetary policy is high. Empirically, we use the monetary uncertainty measure constructed by Bauer, Lakdawala, and Mueller (2022) to test the above implications and find supporting evidence for our model.

Our model is set in continuous time and we focus on the minimum state variable Markov equilibrium. To highlight our new interpretation of the Fed information effect, we stay as close as possible to the textbook New Keynesian model but use fully global solutions to characterize the time-varying steady state of the economy. In our model, Fed type is a persistent state variable modeled as a two-state Markov chain. Beliefs about Fed type is the results of Bayesian learning and is obtained through a filtering problem. They affect aggregate price level because firms' pricing decisions are forward looking. The global solution approach allows us to characterize the nonlinear dynamics uncertainty and study its impact on monetary policy outcomes.

Related literature This paper contributes to the literature on the Fed information effect, which posits that FOMC announcements reveal the central bank's superior information about economic fundamentals. An early discussion of the Fed information effect is Romer and Romer (2000). Several empirical evidences are often interpreted as the Fed information effect. First, about one-third of FOMC announcements since 1990 exhibit positive co-movement between interest rates and stock prices, for example, Jarocinski and Karadi (2020) and Lunsford (2020). Second, professional forecasts of output growth increase and those of unemployment decrease following monetary tightening, see Nakamura and Steinsson (2018) and Campbell, Evans, Fisher, Justiniano, Calomiris, and Woodford (2012). Third, interest rate increases are often accompanied by rising rather than falling prices, for example, Eichenbaum (1992), Sims (1992), and Nakamura and Steinsson (2018).

Different from the above papers, we present an equilibrium model where the monetary authority does not have superior information about economic fundamentals. Rather, monetary policy announcements reflect the Fed's long-run policy objectives. The Fed information effect arises private sector's belief about Fed's type can have persistent impact on economic outcomes, while the impact of monetary policy shock without changes in belief is temporary as in standard New Keynesian models. The economic mechanism of our model therefore differs from the traditional interpretation of the Fed information effect discussed in the above literature as well as the "Fed response to news channel" of Bauer and Swanson (2023a) and Bauer, Pflueger, and Sunderam (2024).

Our paper is related to the literature on high-frequency identification of monetary policy shocks and its impact on the economy. Hanson and Stein (2015), Hanson, Lucca, and Wright (2021), Kekre, Lenel, and Mainardi (2024), and Hillenbrand (2025) focus on the impact of monetary policy on the term structure of interest rates. Bernanke and Kuttner (2005) study the impact of monetary policy shocks on stock market returns. Nakamura and Steinsson (2018), Karnaukh and Vokata (2022), and Bauer and Swanson (2023b) discuss evidence that monetary policy surprises affects economic forecasts of production and employment. Cieslak and Pang (2021) decompose monetary policy surprises into news about growth rates, discount rates and risk premiums.

Our paper builds on the large literature on equilibrium models of the impact of monetary policy on financial markets and the real economy. For comprehensive review of this literature, see Friedman, Hahn, and Woodford (2010). Our paper is more related to recent works that study different channels through which monetary policy surprises affect equity prices, including Drechsler, Savov, and Schnabl (2018), Caballero and Simsek (2020), Pflueger and Rinaldi (2022), Kekre and Lenel (2022), Bianchi, Lettau, and Ludvigson (2022), Caramp and Silva (2024), and Nagel and Xu (2024). Bocola, Dovis, Jørgensen, and Kirpalani (2024) and Bocola, Dovis, Jørgensen, and Kirpalani (2025) also use high-frequency inflation swap data at different horizons to study how monetary policy surprises and learning about policy objectives affect long-run inflation expectations. Gomez Cram, Kung, and Lustig (2024) use high-frequency data from the COVID period to show that Fed large-scale asset purchases in response to unfunded fiscal shocks temporarily support Treasury prices at taxpayers' expense, and that bad news about future fiscal surpluses raises Treasury yields.

This paper also relates to the literature that central banks and the public have disagreement or asymmetric information. For example, Cieslak (2018), Caballero and Simsek (2022), and Sastry (2024) study cases where the Fed and the market disagree about future economic fundamentals, such as the unemployment rate and aggregate demand. Our paper differs by focusing on the case in which market agents do not observe the central bank's long-run type directly and must update beliefs from observed monetary policy surprises using Bayes' rule. In related work, Gomez Cram, Kung, et al. (2025) uses high-frequency Treasury yield responses to CBO cost estimates in a model with learning to show that investors update beliefs about the U.S. fiscal stance and the fraction of deficit news that is expected to be unbacked.

The rest of the paper is organized as follows. We first summarize several stylized facts that motivated the study of Fed information effect in Section 2. We describe a continuous-time New Keynesian model with learning in Section 3. We illustrate our model solutions using policy functions and impulse response functions in Section 4. Section 5 provides empirical evidence for the learning mechanism and Section 6 concludes.

2 Stylized facts on the Fed information effect

We start by summarizing and documenting several stylized facts related to the Fed information effect.

1. Surprise interest rate reductions can be associated with either positive or negative stock market responses. In textbook New Keynesian models, surprise interest rate reductions raises output and, at the same time, reduces discount rate. As a result, the fact that surprise interest rate reductions raise stock market valuation is a robust implication of the standard New Keynesian models. In the data, however, stock market can respond both positively or negatively to interest rate reductions. Updating the evidence of Jarocinski and Karadi (2020) in Figure 1, we plot surprise changes in the short rate, as measured by the innovations in the interest rate futures contact over 30-minute FOMC announcement windows on the horizontal axis and the returns on the S&P500 index during the same interval on the vertical axis. Standard New Keynesian models imply that all data points should be in the II and IV quadrants. However, roughly 30% of the time, stock market return and short term interest rate change move in the same direction over short FOMC announcement windows.

   The above observation led Jarocinski and Karadi (2020) to decompose monetary policy shocks into two components, monetary policy shocks and central bank information shocks. Using their decomposition, we document two additional facts.

2. Identified central bank information shocks are positively correlated with stock market returns, growth rate forecast revisions [2], and inflation forecasts.

[2] Bu, Rogers, and Wu (2021) regressed Blue Chip forecast revision on the central bank information shocks

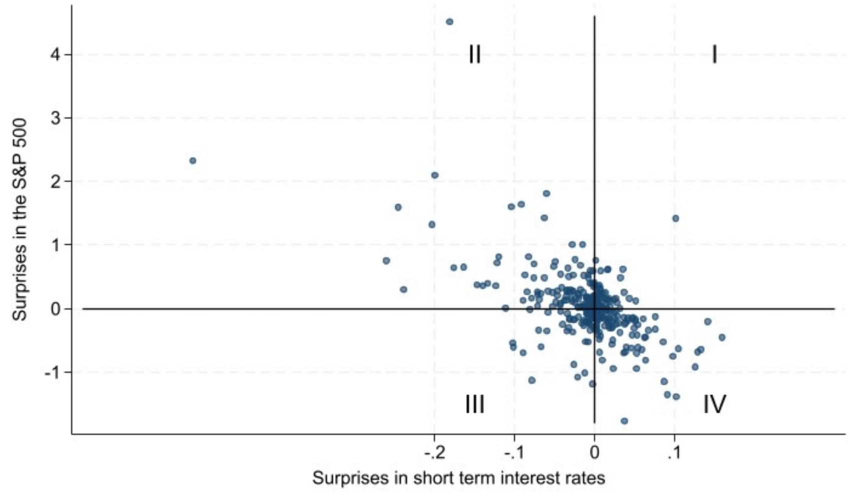


Figure 1: Policy surprises and stock returns around FOMC announcements

Notes: This figure plots the relationship between monetary policy surprises and S&P 500 returns in a 30-minute window around FOMC announcements from 1988 to 2023. The horizontal axis shows surprises in the policy indicator, measured as the first principal component of surprises in interest rate derivatives with maturities from 1 month to 1 year (including federal funds futures and Eurodollar futures), where positive values indicate unexpected monetary tightening. The vertical axis shows percentage changes in the S&P 500 stock index. Data sources: Monetary policy surprises and S&P 500 data from Jarociński and Karadi (updated through 2023).

Surprise interest rate increases associated with central bank information shocks, by construction, is positively correlated with stock market returns. They are also associated with rises in inflation expectations. To measure the impact of different components of monetary policy shocks on inflation expectations, we regress daily price changes in inflation swaps with different maturities on the two components of monetary policy shocks, conventional monetary policy shocks and central bank information shocks, as constructed by Jarocinski and Karadi (2020). Figure 2 reports the regression coefficients on conventional monetary policy shocks (solid line) and central bank information shocks (dashed line), together with one-standard deviation bands. An increase in nominal rate as a conventional monetary policy shock is associated with declines in inflation expectations, as predicted by standard New Keynesian models, although the regression coefficient is not significant. More important, an increase in nominal rate, as a central

from Jarocinski and Karadi (2020), and find that survey respondents revise their output forecast upwards after positive central bank information shocks (Table 4 of Bu, Rogers, and Wu (2021)) bank information shock, is associated with strong and positive responses in inflation expectations.

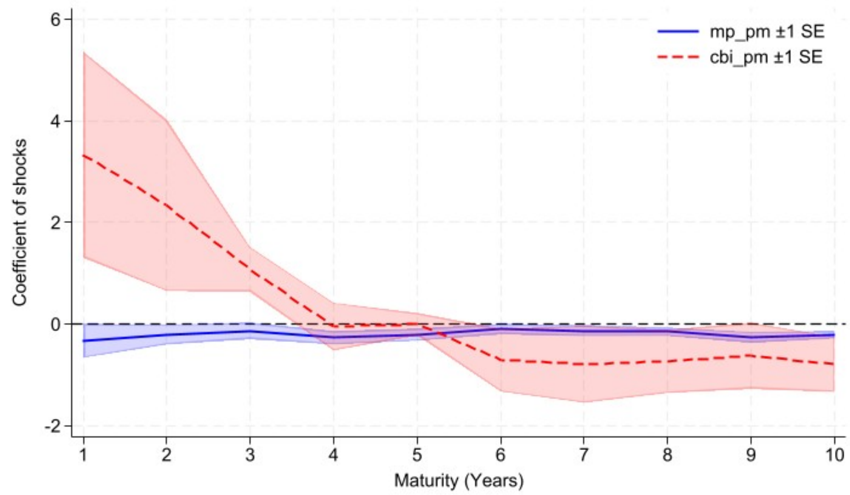


Figure 2: Inflation swap responses to monetary policy vs. information shocks

Notes: This figure plots the estimated coefficients from regressions of inflation swap rate changes on monetary policy shocks (mp_pm, shown in blue solid line) and central bank information shocks (cbi_pm, shown in red dashed line) across different maturities. The horizontal axis shows the maturity of inflation swaps in years (1 to 10 years). The vertical axis shows the coefficient estimates. Shaded areas represent 68% confidence intervals (approximately one standard error). The regressions use HAC robust standard errors with bandwidth of 4. Data span from 2004 to 2023. Inflation swap data are from Bloomberg. Monetary policy shocks and central bank information shocks are decomposed shocks measured in a 30-minute window around FOMC announcements from Jarocinski and Karadi (2020). Detailed regression results including coefficient estimates, standard errors, and difference tests are reported in Appendix Table 5.

3. The yield curve responds to both conventional monetary policy shocks and Fed information shocks, but the response to central bank information shocks is stronger across all maturities.

   Figure 3 presents regression coefficients of high-frequency changes in 2-year, 5-year, 10-year, and 30-year Treasury yields measured around the FOMC announcement windows on the conventional monetary policy shocks (solid line) and those on central bank information shocks (dashed line). Both types of shocks raise yields at all maturities, but the effect of central bank information shocks is consistently stronger than that of monetary policy shocks.

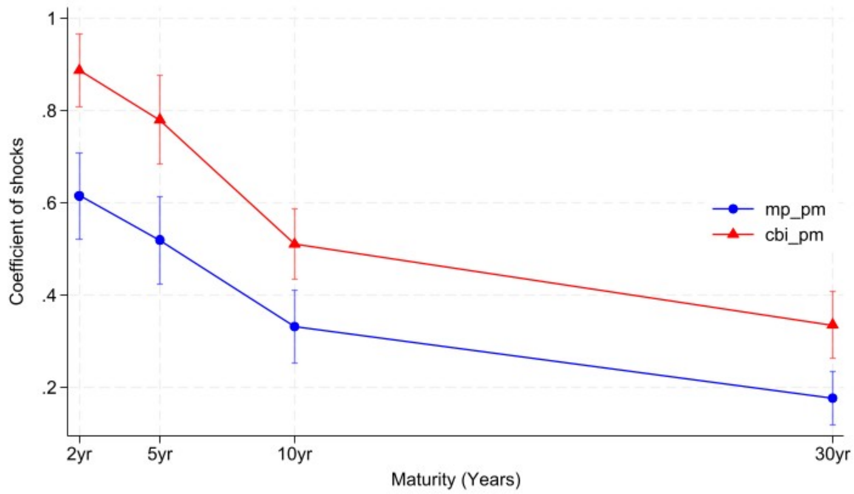


Figure 3: Treasury yield responses to monetary policy vs. information shocks

Notes: This figure plots the estimated coefficients from regressions of Treasury yield changes on monetary policy shocks (mp_pm, shown in blue circles) and central bank information shocks (cbi_pm, shown in red triangles) across different maturities. The horizontal axis shows the maturity of Treasury securities (2-year, 5-year, 10-year, and 30-year bonds). The vertical axis shows the coefficient estimates. Error bars represent 68% confidence intervals (approximately one standard error). The regressions use HAC robust standard errors with bandwidth of 4. Data span from 1990 to 2023. Monetary policy shocks and central bank information shocks are decomposed shocks measured in a 30-minute window around FOMC announcements from Jarocinski and Karadi (2020). High-frequency Treasury yield data and shock series are from Bauer and Swanson (2023b). Detailed regression results including coefficient estimates, standard errors, and difference tests are reported in Appendix Table 6.

The fact that surprise interest hikes may be associated with increases in stock market valuation and growth rate forecasts is puzzling. This pattern leads researchers as such as Nakamura and Steinsson (2018) to conclude the "Fed information effect", that is, the monetary authority has superior information about economic fundamentals. When they announce a contractionary monetary policy, the market reacts positively because it interprets the policy as revealing a stronger than expected macroeconomic fundamentals. Direct evidence for Fed's superior information about macroeconomic fundamentals, however, is rare. In this paper, we present a simple variation of the standard New Keynesian model, where the monetary authority has superior information about its own long-run policy objectives. We show that this mechanism alone can explain the above facts without assuming Fed's superior information about macroeconomic fundamentals.

3 A New Keynesian model of Fed type

In this section, we set up a New Keynesian model where Fed type is driven by a persistent state variable modeled as a Markov process. We stay as close as possible to the textbook New Keynesian setup so that we can focus on the implications of the key mechanism of learning and uncertainty about Fed's long-run policy objective.

3.1 Model setup

Households Time is continuous and infinite. We assume that the representative household's preference is:

$$E\left[\int_0^\infty e^{-\rho t}\left(\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{L_t^{1+\zeta}}{1+\zeta}\right)dt\right], \tag{1}$$

where C_t is consumption of final output, and L_t is labor supply. $\gamma > 0$ is the risk aversion and $\zeta > 0$ is the inverse Frisch elasticity of labor supply. The preference is the standard additively separable expected utility used in textbook New Keynesian models.

The representative consumer seeks to maximize life-time expected utility subject to the budget constraint:

$$E\left[\int_0^\infty \Lambda_t\left(C_t - w_t L_t\right)dt\right] \leq 0,$$

where Λ_t is the stochastic discount factor that prices time-t real consumption into time-0 consumption units, and w_t is real wage at time t.

Firms' production and pricing decisions The final output Y_t is produced using a continuum of intermediate inputs via a CES production function $Y_t = \left[\int_0^1 y_{i,t}{}^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}$, where y_{it} denotes output of differentiated commodity variety i. The parameter $\eta > 1$ denotes the elasticity of substitution between different varieties. The final goods producers are fully competitive and their demand for individual variety i is given by

$$y_{it} = (\frac{p_{i,t}}{P_t})^{-\eta} Y_t, \tag{2}$$

where $p_{i,t}$ is the price for commodity i, and P_t is the price of general consumption Y_t. Because the production technology is constant returns to scale, the competitive final goods producer's equilibrium profit is zero, and this in turn implies that $p_{i,t}$ and P_t must be related by

$$P_t = \left[\int p_{i,t}^{1-\eta} dt \right]^{\frac{1}{1-\eta}}. \tag{3}$$

There are a continuum of firms or intermediate good producers indexed by i. Firm i produces good i using a constant returns to scale technology with labor as the only input, $y_{it} = Al_{it}$. As is standard in the New Keynesian setup, firms produce on monopolistically competitive markets to maximize the present value of profit. Given $p_{i,t}$ and P_t, if the time t output of firm i is $y_{i,t}$, firm profit, measured in real consumption terms, is given by $\frac{p_{i,t}}{P_t} y_{i,t} - w_t \frac{y_{i,t}}{A}$. Using the demand function (2), real profit is a function of $\frac{p_{i,t}}{P_t}$, $\Psi\left(\left.\frac{p_{it}}{P_t}\right| w_t, Y_t\right)$, taking w_t and Y_t as given:

$$\Psi\left(\left.\frac{p_{it}}{P_t}\right| w_t, Y_t\right) = \frac{p_{it}}{P_t} \times \left(\frac{p_{it}}{P_t}\right)^{-\eta} Y_t - w_t \times \frac{1}{A} \left(\frac{p_{it}}{P_t}\right)^{-\eta} Y_t.$$

Firms pricing decisions are forward looking and subject an adjustment cost, as in Rotemberg (1982). Let π_{it} be the rate of price change at time t, then the law motion of p_{it} is,

$$dp_{i,t} = p_{i,t}\pi_{i,t} dt.$$

The present value of all future profit for firm i at time t is

$$\max_{\{\pi_{i,t}\}_{t=0}^{\infty}} E_t \left[\int_0^{\infty} \Lambda_{t,t+s} \left\{ \Psi\left(\left.\frac{p_{t+s}}{P_{t+s}}\right| w_{t+s}, Y_{t+s}\right) - h\left(\pi_{i,t+s}\right) Y_{t+s} \right\} ds \right], \tag{4}$$

where $\Lambda_{t,t+s}$ is stochastic discount factor given in (1). The function h is a strictly increasing and strictly convex cost function of price adjustment. We set $h(\pi) = \frac{h_0}{2}\pi^2$, where h_0 is a parameter that governs the magnitude of cost of adjustment as in Rotemberg (1982). The total cost of price adjustment at time t is proportional to total output at time t, Y_t.

Market clearing Goods market clearing requires that total consumption plus total cost of price adjustment must equal total output:

$$C_t + \int h\left(\pi_{i,t}\right) Y_t di = Y_t.$$

In addition, labor market clearing implies that, for all t,

$$\int l_{i,t} di = L_t.$$

Monetary policy rule Individual firm's pricing decisions implies a dynamics of aggregate price index through (3). We denote the rate of change of the aggregate price index as $\pi_t = \frac{dP_t}{P_t}$, that is, π_t is aggregate inflation. Give inflation π_t, the monetary authority follows the Taylor rule when setting the nominal interest rate:

$$i_t = \phi \pi_t + \nu_t, \tag{5}$$

where ν_t is a diffusion process with

$$d\nu_t = a\left(\theta_t - \nu_t\right) dt + \sigma_\nu dB_{\nu,t}. \tag{6}$$

Here, $B_{\nu,t}$ is a standard Brownian motion and θ_t represents the long-run target rate. The above is a continuous-time version of a standard New Keynesian model, for example in Galí (2015). Equation (6) implies that interest rate policy are subject to temporary shocks, $B_{\nu,t}$ but have a tendency to converge to θ_t in the long-run. We can think of $B_{\nu,t}$ as transitory preference shocks from the Fed and θ_t as the long-run target rate. To model learning about Fed's policy target, we now turn to the specification of the dynamics of θ_t.

Learning about Fed type We assume θ_t is a two-state MC with state space $\{\theta_H, \theta_L\}$ and an infinitesimal generator $\begin{bmatrix} -\kappa_H & \kappa_H \\ \kappa_L & -\kappa_L \end{bmatrix}$. Intuitively, the transition matrix of θ_t over a small interval Δ is $\begin{bmatrix} 1-\kappa_H\Delta & \kappa_H\Delta \\ \kappa_L\Delta & 1-\kappa_L\Delta \end{bmatrix}$. We assume $\theta_H > \theta_L$ so that θ_H corresponds to a hawkish Fed with a higher long-run interest rate target and θ_L represents a dovish Fed. A hawkish Fed may set temporarily low interest rates due to preference shocks but will eventually raise rates aggressively in the long run.

We assume that the true type of the Fed, θ_t is not observable. The private sector measures monetary shocks from each FOMC meetings and observe the history of Fed's interest rate decisions as well. Given the history of interest rate i_t and inflation π_t, it can back out the history of ν_t using Equation (5) but not that of θ_t. Rational investors will form Bayesian beliefs about θ_t using the history of ν_t. Thanks to the assumption of two-sate Markov chain, the posterior mean of θ_t, which we will denote as $\hat{\theta}_t$, is enough to summarize the posterior distribution and serve as a state variable in the Markov equilibrium.

Using Theorem 9.1 in Liptser and Shiryaev (2001), we can write the law of motion of $\hat{\theta}_t$ as:

$$d\hat{\theta}_t = (\kappa_H + \kappa_L)\left(\bar{\theta} - \hat{\theta}_t\right)dt + \left(\theta_H - \hat{\theta}_t\right)\left(\hat{\theta}_t - \theta_L\right)\frac{a}{\sigma_\nu}d\tilde{B}_t, \tag{7}$$

where $\bar{\theta}$ is the steady-state mean of θ, $\bar{\theta} = \frac{\kappa_L\theta_H + \kappa_H\theta_L}{\kappa_L + \kappa_H}$, and the innovation process $d\tilde{B}_t$ is defined as:

$$d\tilde{B}_t = \frac{1}{\sigma_\nu}\left[d\nu_t - a\left(\hat{\theta}_t - \nu_t\right)dt\right].$$

Theorem 9.1 in LS implies that $\tilde{B}_t$ is a Brownian motion with respect to investors' information set. Using the definition of the innovation process above, we can rewrite the monetary shock process as an adapted process with respect to investors' information set:

$$d\nu_t = a\left(\hat{\theta}_t - \nu_t\right)dt + \sigma_\nu d\tilde{B}_{\nu,t}. \tag{8}$$

3.2 Markov Equilibrium

In this paper, we focus on Markov equilibriums where equilibrium quantities and prices are functions of a minimum set of Markov state variables.[3] Because all the intermediate firms have the same productivity and face the same production and price setting decisions, we also impose symmetry which requires that all firms make the same production and price setting choices.

Formally, a symmetric Markov equilibrium consists of

1. a wage function $w\left(\hat{\theta},\nu\right)$, an inflation function $\pi\left(\hat{\theta},\nu\right)$, real interest rate $r\left(\hat{\theta},\nu\right)$, and an equilibrium stochastic discount factor $\Lambda\left(\hat{\theta},\nu\right)$,

2. a labor supply function $L\left(\hat{\theta},\nu\right)$, a consumption policy function $C\left(\hat{\theta},\nu\right)$, and an

[3]Equilibriums are typically not unique in New Keynesian models. As we show by construction, minimum state variables select a unique equilibrium in our setup. Angeletos and Lian (2023) provide a motivation for minimum state variables.

output function $Y\left(\hat{\theta},\nu\right)$,

such that the following conditions are met:

1. The equilibrium stochastic discount factor, $\Lambda\left(\hat{\theta},\nu\right)$ is consistent with households intertemporal optimization over consumption.

$$\Lambda\left(\hat{\theta},\nu\right)=C\left(\hat{\theta},\nu\right)^{-\gamma} \tag{9}$$

2. Given the real wage function, $L\left(\hat{\theta},\nu\right)$ satisfy the intra-temporal optimality condition for labor supply:

$$\frac{L\left(\hat{\theta},\nu\right)^{\zeta}}{C\left(\hat{\theta},\nu\right)^{-\gamma}}=w\left(\hat{\theta},\nu\right). \tag{10}$$

3. Given $\pi\left(\hat{\theta},\nu\right)$ and the law of motion of state variables (7), and (8), we can construct the aggregate prices index using $dP_t=P_t\pi\left(\hat{\theta}_t,\nu_t\right)dt$. Given the dynamics of P_t, and the equilibrium wage and output functions, $w\left(\hat{\theta},\nu\right)$ and $Y\left(\hat{\theta},\nu\right)$, intermediate firms i chooses their symmetric forward-looking optimal price $\pi_i=\pi_i\left(\hat{\theta},\nu\right)$.

4. Intermediate output is symmetric across firms as well:

$$Y\left(\hat{\theta},\nu\right)=AL\left(\hat{\theta},\nu\right). \tag{11}$$

5. Aggregate resource constraint is satisfied, that is,

$$C\left(\hat{\theta},\nu\right)+h\left(\pi\left(\hat{\theta},\nu\right)\right)Y\left(\hat{\theta},\nu\right)=Y\left(\hat{\theta},\nu\right), \tag{12}$$

6. The real interest rate is derived from the stochastic discount rate via the no-arbitrage relationship:

$$r\left(\hat{\theta},\nu\right)=-E\left[\frac{d\Lambda(\hat{\theta},\nu)}{\Lambda(\hat{\theta},\nu)}\right].$$

7. The nominal interest, $i=r\left(\hat{\theta},\nu\right)+\pi\left(\hat{\theta},\nu\right)$ satisfies Taylor rule (5).

The above conditions can be simplified to derive a set of functional equations that can be used to solve for the equilibrium.

4 Model implications

Table 1: Parameter Values of Model Solution

Parameter	Description	Value
ρ	discount rate	0.015
A	productivity	1
θ_L	low long-run rate	0
θ_H	high long-run rate	0.04
$\bar{\theta}$	middle long-run rate	0.02
γ	risk aversion	1
η	substitution elasticity of intermediate goods	3
ζ	inverse of labor elasticity	0
κ_H	transition probability from high to low regime	0.25
κ_L	transition probability from low to high regime	0.25
ϕ	Taylor coefficient	3
a	mean-reverting rate of transitory shocks	0.99
h_0	price adjustment cost	5

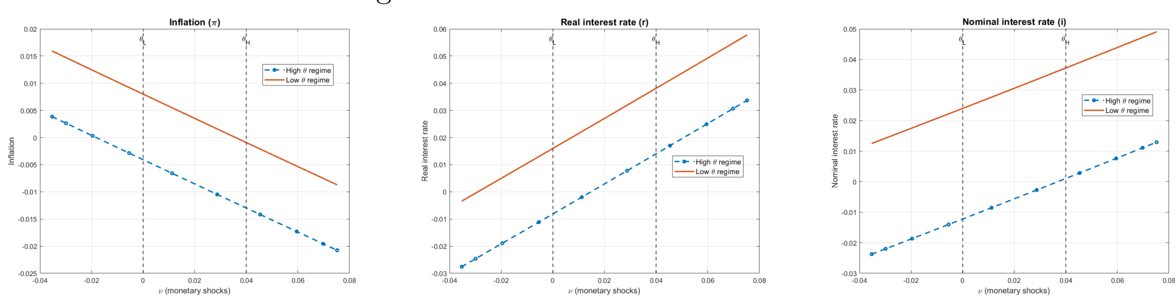
Notes: Parameter values used in the model solution.

In this section, we present model solutions by presenting policy functions and impulse response functions. Our parameter values are standard based on textbooks and are listed in Table 1. It is worth noting that we choose the values of κ_H and κ_L such that the average duration of the long-run rate regime is 4 years, equal to the tenure of a Fed president appointment.

Using these parameter values, we first focus on the case where Fed type θ is observable.

Observable Fed type To illustrate the implications of the model, we first plot the impact of monetary policy shocks on inflation (left panel), real interest rate (middle panel), and the nominal interest rate (right panel) in Figure 4. The horizontal axis is the value of the short-rate shock, ν. The blue dashed lines are policy functions for $\theta_t = \theta_H$ and the red solid lines are policy functions for $\theta_t = \theta_L$. We make two observations. First, keeping the policy regime θ fixed, a negative shock to Taylor rule, ν, raises inflation and lowers the real interest rate and the nominal rate. This is the classical New Keysian channel of monetary shocks. To understand this, note that the Taylor rule implies $r = (\phi - 1)\,\pi + \nu$. In a flexible price equilibrium, a negative shock to ν cannot affect r and must imply a positive innovation in inflation π. Under sticky price, adjustment in π is costly and therefore not enough to completely offset the change in ν. As a result, real rate r must go down to keep the relationship $r = (\phi - 1)\,\pi + \nu$ satisfied. Because the nominal rate $i = r + \pi$, a small increase in π combined with a reduction in r implies a lower nominal rate as shown in the right panel of Figure 4.

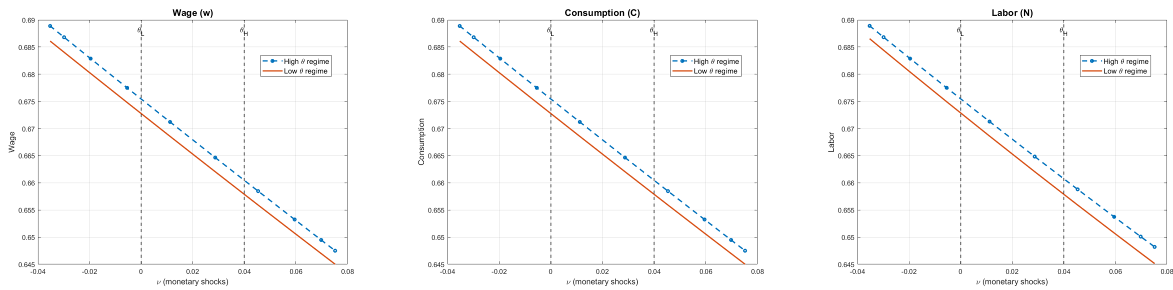
Figure 4: Inflation and interest rates



*Notes:*This figure plots the policy functions of inflation (left panel), real interest rate (middle panel), and nominal interest rate (right panel) as a function of state variables $\{\theta, \nu\}$. The horizontal axis is the monetary policy shock ν. The blue dashed lines are policy function for the high-target rate regime, θ_H and the red solid lines are the policy functions for the low target rate regime, θ_L.

Second, keeping monetary policy variable ν fixed, a reduction in the long-run target rate raises inflation, raises the nominal rate as well as the real rate. A reduction in long-run interest rates, that is, a switch from θ_H to θ_L, raises expectation about future inflation. Because firms' pricing decisions are forward looking, a higher inflation expectation immediately translates into a higher current inflation. Given that ν is kept fixed, the Taylor rule $r = (\phi - 1)\,\pi + \nu$ implies that a higher π must be associated with a higher real rate r. Finally, $i = r + \pi$ must increase as both r and π increases following a reduction in θ. In contrast to the negative temporary shock ν, which is associated with a lower nominal and real rate, a negative shock to the long-run target rate causes nominal and real rates to move opposite directions.

Figure 5: Real impact of monetary policy



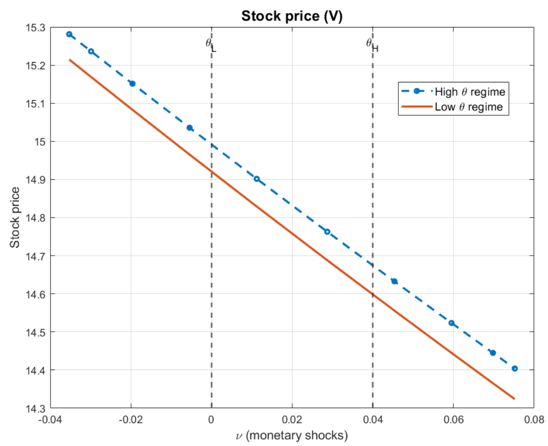
Notes: This figure plots the policy functions of real wage (left panel), consumption (middle panel), and labor supply (right panel) as a function of state variables $\{\theta, \nu\}$. The horizontal axis is the monetary policy shocks ν. The blue dashed lines are policy functions for the high-target rate regime, θ_H and the red solid lines are the policy functions for the low target rate regime, θ_L.

We now turn to the implications of the model on the real side of the economy. We plot the policy functions for real wage, $w(\theta, \nu)$, consumption, $C(\theta, \nu)$, and labor supply, $L(\theta, \nu)$ in Figure 5. The horizontal axis is the value of short-rate shock, ν. The blue dashed lines are policy functions for $\theta_t = \theta_H$ and the red solid lines are policy functions for $\theta_t = \theta_L$. For a fixed policy regime θ, lower values of ν are associated with higher levels of real wage, consumption, and labor supply. This is the standard New Keynesian effect of monetary policy: negative shocks to the Taylor rule raise the real wage, boost labor supply, and result in an increase in aggregate output.

Now consider a decrease in θ, that is, a shift of policy regime from θ_H to θ_L, by keeping the value of ν constant. In this case, a reduction of the long-run target rate is associated a a lower real wage, a lower labor supply, and a lower consumption level. A lower target rate signals a lower long-run interest rate and a higher long-run inflation. Keeping current inflation level constant, this must be associated with a lower real wage and hence a lower labor supply and aggregate output.

The above patterns of the policy function highlights the different, and often apposite implications of shocks to current-period inflation and shocks to long-run inflation expectations. Keeping long-run inflation expectation, which is captured by the state variable θ, constant, an increase in inflation is expansionary and raises real wage, labor supply, consumption and output. This is the standard New Keynesian channel of monetary policy. However, keeping the current level of ν constant, a raise in future inflation, through changes in θ, also raises equilibrium inflations, but it is a contractionary shock. It raise current level of inflation because private sector's price adjusting behavior is forward looking: anticipating a risk of inflation in the future, firms raise their prices today to avoid costly sudden increases of inflation due to the convex price adjustment cost. It is a contractionary shock, because firms have a higher incentive to inflate prices for a given level of real wage, and as a result, equilibrium real wage will be lower and so is employment and total output.

Figure 6: Policy Function with Regime Switch and Perfect Information



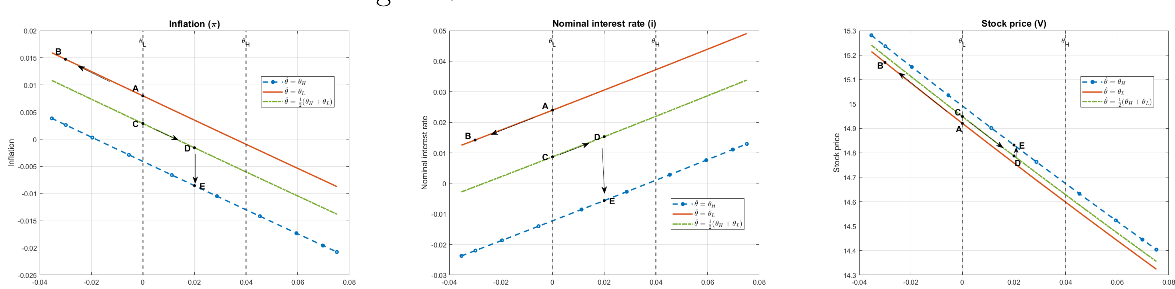
Notes: This figure plots the policy functions of equity price as a function of state variables $\{\theta, \nu\}$. The horizontal axis is the monetary policy shocks ν. The blue dashed lines are policy function for the high-target rate regime, θ_H and the red solid lines are the policy functions for the low target rate regime, θ_L.

Finally, we plot the model's implications on asset prices in Figure 6. For a given regime, the equity value of a firm is a decreasing function of ν. That is, a negative shock to Taylor rule lowers the real interest rate and raises equity market valuation. This is consistent with the standard New Keynesian effect of monetary policy shocks. Keeping the value of ν constant, a reduction in the long-run target rate lowers equity valuations. From Figure **??**, a reduction in the target rate lowers output and therefore dividend payout. From Figure 4, it raises the discount rate. The two forces combined together to lower equity valuation. Again, reductions in the long-run target rate have the exact opposite effect to the those in the short-term interest rates.

In summary, shocks to the short rate and shock to the target rate (or changes in policy regime) have opposite effect on real quantities and equity market valuations. A reduction in the short rate is a classical monetary shock in NK models. As an expansionary shock, it reduces unemployment, boosts real output, and raises equity market valuations. In contrast, a change in the target rate, θ, affects investors' belief about policy regime. An reduction in the target rate is a contractionary shock: it raises inflation expectations, lowers employment, and depresses equity valuations.See Appendix A.2 for more of analytical explanation from the model on this part.

Learning about Fed type In this section, we illustrate the implications of our full model where Fed's policy regime is not directly observable and has to be inferred from past actions of monetary policy. In this case, an innovation in the monetary policy shock ν has two effects. First, it affects the current inflation and nominal interest rate through the Taylor rule (5), just like in conventional New Keynesian models. When uncertainty about Fed's policy regime is low, this is the dominating effect. Second, it impacts the private sector's belief about Fed's type. As we have shown in the previous section, this information channel affects inflation expectations and often has opposite effects as the standard New Keynesian channel.

Figure 7: Inflation and interest rates



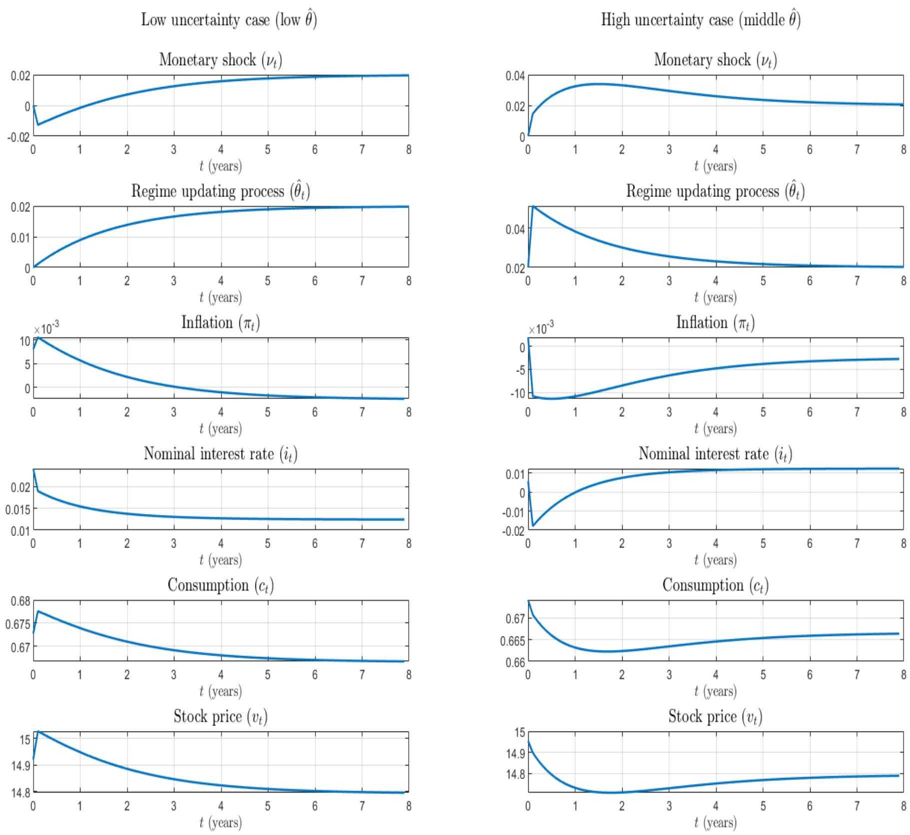
To illustrate the different impact of the two types of monetary policy shocks, in Figure 7, we assume that the current-period value of ν is $\nu_0 = \theta_L$ and use the policy functions to illustrate two different paths of shocks to ν_0 for different initial values of the state variable $\hat{\theta}_0$. Consider the first scenario where $\hat{\theta} = \theta_L$. In this case, investors have little uncertainty about the value of θ_0 and shocks to ν_0 does not affect the belief about $\hat{\theta}_0$. As a result, a negative shock to ν_0 moves the inflation (left panel of Figure 7) upwards along the red solid line ($\hat{\theta} = \theta_L$) from point A to point B. As in standard New Keynesian models, a reduction in ν_0 raises inflation and lowers the nominal rate (middle panel of 7). Correspondingly, stock price from point A to B in the right panel. Because this shock lowers the nominal interest rate, it will be measured as an expansionary shock: it raises real wage, consumption, output, and stock market valuation.

We plot the above path of shocks in the impulse response functions on the left panel in Figure 8. In the top figure, a negative shock moves ν_t downwards from θ_L immediately, before it recovers in the next period and slowly converges back to the steady state of $\bar{\theta} = \frac{\lambda_L \theta_H + \lambda_H \theta_L}{\lambda_L + \lambda_H}$. In the second figure on the left panel, the belief about $\hat{\theta}_t$ does not respond to the shock because the prior belief, $\hat{\theta}_0 = \theta_L$ focuses on θ_L with little uncertainty. It merely converges to steady state of $\bar{\theta}$ in the long run. Consistent the policy functions in Figure 7, the nominal rate drops immediately, and inflation rises immediately, and converges to the steady state in the long run. Steady-state inflation is lower then its initial level, because the steady state level of v_t is $\bar{\theta}$, higher than the initial level of $\nu_0 = \theta_L$. The rest of the impulse response functions all follow the same pattern: real wage, consumption and stock price all increase upon impact and converge to a lower steady state in the long-run as v_t converges to $\bar{\theta}$.

Consider now the second case where $\nu_0 = \theta_L$ but $\hat{\theta}_0 = \frac{1}{2}(\theta_H + \theta_L)$, represented by point C in Figure 7. In our setup, $\hat{\theta}_t \in [\theta_L, \theta_H]$. The posterior probability is $P(\theta_t = \theta_H | t) = \frac{\hat{\theta} - \theta_L}{\theta_H - \theta_L}$, and the posterior variance of agents' belief is $Var[\theta_t | t] = \frac{\hat{\theta}_t - \theta_L}{\theta_H - \theta_L}\left(\theta_H - \hat{\theta}_t\right)^2 + \frac{\theta_H - \hat{\theta}_t}{\theta_H - \theta_L}\left(\theta_L - \hat{\theta}_t\right)^2$. Clearly, posterior variance is minimized at $\hat{\theta} = \theta_H$ or $\hat{\theta} = \theta_L$ and maximized at $\hat{\theta} = \frac{1}{2}(\theta_H + \theta_L)$. At $\hat{\theta}_0 = \frac{1}{2}(\theta_H + \theta_L)$, the prior belief about θ_0 is highly uncertain, and the posterior belief will be highly sensitive to variations in ν. In this case, shocks to ν will lead to significant updates of the posterior belief about θ.

Consider a positive shock to ν_0: it moves the nominal rate from point C to point D along the dash-dotted line if the posterior belief, $\hat{\theta}$, were kept constant. However, Bayesian updating requires investors to revise their posterior belief upwards from $\hat{\theta} = \bar{\theta}$, represented by point E on the dashed line in the figure, where for the purpose of illustration, we assume that the belief revision moves $\hat{\theta}$ all the way to θ_H. Investors are uncertain about whether a positive shock to ν is originated from the temporal Brownian motion shock $dB_{\nu,t}$ or is due to a persistent change in the interest rate target θ. As a result of Bayesian learning, they rationally assign a non-trivial probability to a regime change in θ and update belief from $\bar{\theta}$

Figure 8: Impulse responses functions of prices and quantities



to θ_H. On the left panel of Figure 7, a tightening shock lowers inflation from C to D due to the traditional New Keynesian effect, and, additionally, from D to E because investors rationally lower price further due to the anticipation of a persistent shift to a high interest rate regime represented by θ_H.

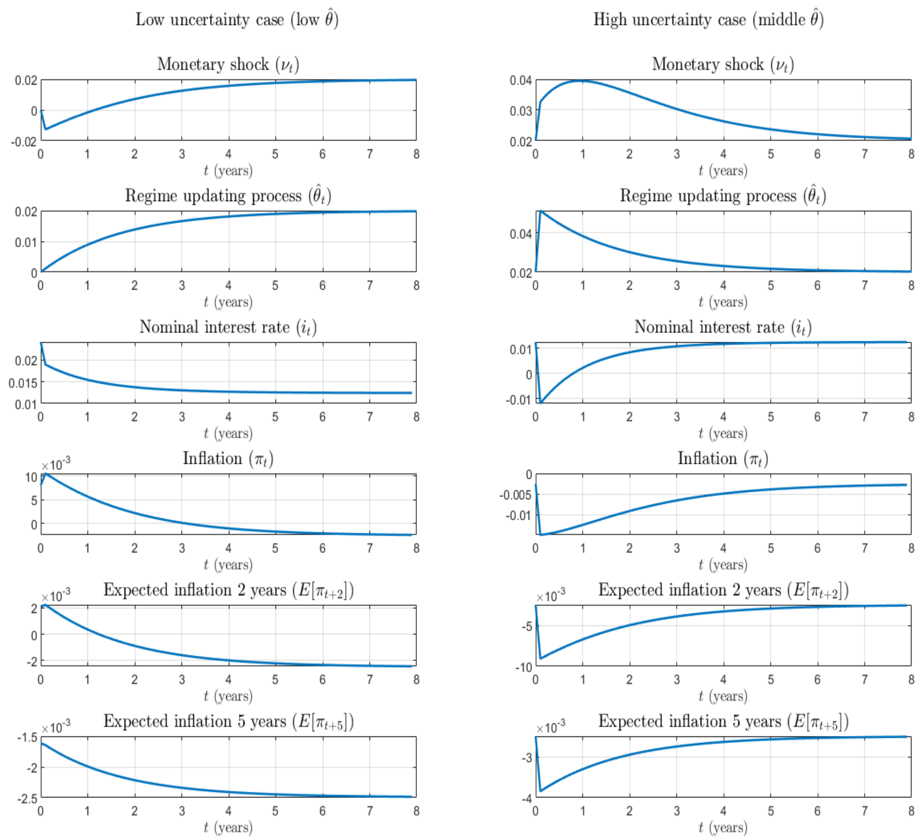
This pattern of inflation dynamics translates into a drop in nominal rate in response to an increase in ν. The standard New Keynesian mechanism increases the nominal rate from C to D along the dash-dotted line ($\bar{\theta}$) if belief were kept constant. However, due to Bayesian updating, nominal rate drops from D to E. The Bayesian updating effects dominates the traditional New Keynesian effect for two reasons, First, anticipating a tightening monetary policy regime, investors lower expectation about future consumption and as a result, real rate drops. This effect is absent in standard New Keynesian models, where reductions in real rate must be accompanied by simultaneous increases in consumption, because monetary policy does not affect the long-run outcome of the economy. Second, as explained above, a tightening shock also lowers inflation. Both effects work in the same direction, and as a result, an increases in ν leads to a reduction, not an increase, in the nominal rate.

At the same time, stock price also moves down from C to D and up from D to E due the anticipation of a lower inflation in the future. The change in stock price from D to E is relatively small, so that the New Keynesian effect dominates for stock price and an increasing ν results in a reduction of stock market valuation. The relative small magnitude of the Bayesian updating impact on stock price can be explained by two offsetting effects. First, an increase in the posterior belief $\hat{\theta}$ raises real wage and total output keeping the current value of ν_t fixed, which tend to raise stock market valuation. Second, a higher posterior belief of $\hat{\theta}$ also implies that the economy is more likely to converge to a steady state with a persistently higher ν, because θ_t is the long-run value of ν_t. A steady state with a higher long-run interest rate is associated with lower long-run output. Because stock market valuation is forward looking, the increase in stock price due to the first channel of Bayesian updating is relatively small, and second channel reinforces the traditional New Keynesian effect to generate a negative stock price response to the initial shock in ν_0.

In summary, when uncertainty is high, an increase in ν_t leads to a simultaneous reduction in interest rate and a decline in stock market valuation. The empirical methodology of Jarocinski and Karadi (2020) would identify this as a Fed information shock, even though in our model, the Fed does not have superior information about macroeconomic fundamentals..

We illustrate the impulse response to this Fed information shock in the right panel in Figure 8. In the second panel, a positive shock to ν_0 when $\hat{\theta}_0 = \bar{\theta}$ raises $\hat{\theta}$ immediately due to Bayesian updating in contrast to the lack of response of $\hat{\theta}$ on the right panel. The nominal rate drops, because, anticipating a regime change to a lower inflation steady state, the negative response of inflation is much stronger than the traditional New Keynesian channel on the left panel of the figure. Despite the drop in the nominal rate, consumption and stock price both decline upon impact, that is, the nominal interest rate and stock price respond in the same direction to monetary policy shocks, a defining feature of the Fed information effect.

Figure 9: Impulse responses functions of inflation expectations



To illustrate the impact of the two types of shocks on inflation and inflation expectations, we plot the impulse response functions of inflation expectations of various maturities to the two types of shocks in Figure 9. On the left panel, we plot the impulse responses of immediate inflation (fourth panel) and inflation expectations with 2 and 5 years of maturity (fifth panel and sixth panel) with respect a conventional monetary policy shock (i.e. a negative shock to ν_0 when uncertainty about θ is low) on the left panel, and those with respect to a Fed information shock (i.e. a positive shock to ν when the uncertainty about θ is high) on the right panel. In the figure, both types of shocks manifest themselves as a reduction in the nominal rate. On the left panel, the traditional New Keynesian shock induced reduction in the nominal rate is associated with a mild increase in current inflation, but the impact on inflation expectation vanishes quickly as maturity increases to two and five years. This is because of the low persistence of the shocks. On the right panel, however, the Fed information shock induced reduction in nominal rate is associated with a strong immediate decline in inflation and the impact of inflation persists even as maturity increases to five years. This patterns of response of inflation expectation to monetary policy shocks and Fed information shocks is consistent with the stylized facts we document in Section 2.

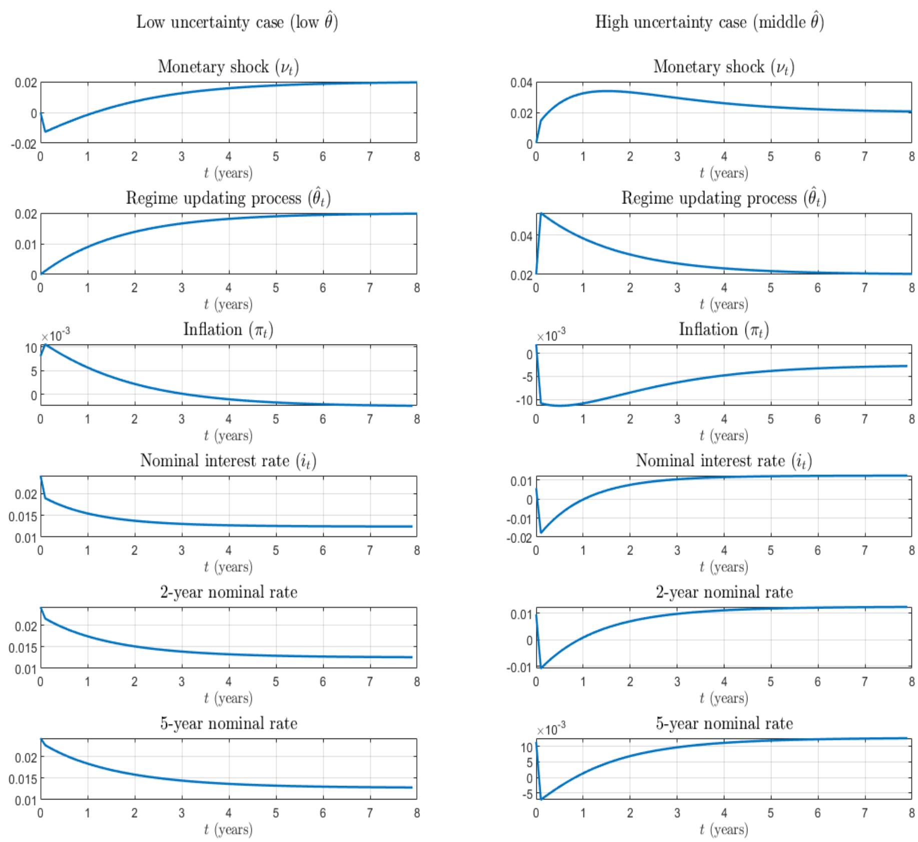
Finally, in Figure 10, we plot the impulse response functions of the term structure of interest rates with respect to a conventional monetary policy shock on the left panel and those with respect to a Fed information shock on the right panel. A conventional monetary policy shock leads to an immediate nominal rate decline, but the magnitude of decline drops significantly at the two-year maturity and almost vanishes at the five-year maturity. By contrast, the Fed information shock results in a larger and more persistent nominal interest decline, because it impacts the long-run steady state of the economy. This pattern is also consistent with the stylized facts we document in Section 2.

5 Empirical evidence and quantitative results

The above analysis implies that when uncertainty about Fed type is low, monetary shocks impact the economy as in textbook New Keynesian models. However, when uncertainty is high, monetary shocks cause quantities and prices to move in opposite directions relative to textbook New Keynesian predictions and can be interpreted as Fed information shocks.

To test these implications, we examine the interaction between monetary policy uncertainty and the impact of monetary policy shocks on asset prices. We measure monetary

Figure 10: Impulse responses functions of nominal yields



policy uncertainty (MPU) following Bauer, Lakdawala, and Mueller (2022) as the conditional standard deviation of the 3-month interest rate implied by Eurodollar options (SOFR options since October 2022), interpolated to a 24-month maturity. Our baseline shock measure is the orthogonalized monetary policy surprise from Bauer and Swanson (2023b). As shown by Cieslak (2018) and Bauer and Swanson (2023a), Fed funds futures price changes measured over high frequency FOMC windows are partly predictable by past macro variables. We follow Bauer and Swanson (2023b) to measure monetary policy surprises by using the component of the high-frequency Fed funds futures price changes that are orthogonal with respect to six macro-financial predictors (denoted MPS_ORTH in Table 2). By construction, this measurement of monetary policy surprise is not predictable by past publicly available information. It allows us to focus on the part of Fed information effect that is not account for by the Fed response to news channel proposed by Bauer and Swanson (2023a).

Our first test is on the Fed information effect on stock prices. Our model predicts that Fed information shocks are more likely to happen when uncertainty about the longer-term interest rate target is high. When monetary policy uncertainty is low, our model behaves like a standard New Keynesian model and nominal rates reductions are associated with stock price appreciations. When monetary policy uncertainty is high, the Fed information effect dominates and nominal rate reductions are associated with stock market declines. We test this implications of our model by using the following regression specification:

$$R_{t,t+\Delta} = \beta_0 + \beta_I \, \text{MPS\_ORTH}_t + \beta_S \, \text{MPU}_{t-1} + \beta_{\text{Inter}} \left(\text{MPS\_ORTH}_t \times \text{MPU}_{t-1}\right) + \epsilon_t, \quad (13)$$

where $R_{t,t+\Delta}$ is the S&P 500 return over the 30-minute FOMC announcement window and MPU_{t-1} is monetary policy uncertainty measured the day before the monetary policy announcement.

Under the null hypothesis of the model, interest rate rises are associated with stock market declines when uncertainty is low. As a result, $\beta_I < 0$. The negative relationship between interest rate surprise and stock market price changes becomes positive when uncertainty about monetary policy is elevated, that is, $\beta_{Inter} > 0$. We report our regression results in Table 2. Column (1) reports the baseline relationship: a one percentage point contractionary shock reduces stock returns by 5.34 percentage points unconditionally. Column (2) adds MPU and its interaction with the policy shock. The coefficient on the interaction term, β_{Inter}, is positive (3.43) and statistically significant at the 5% level. These results are consistent with our model's predictions.

Table 2: S&P 500 Returns and Monetary Policy Uncertainty

	(1) Baseline	(2) + MPU and interaction
MPS_ORTH	-5.34^{***}	-6.12^{***}
	(−6.11)	(−5.83)
MPU_{t-1}		-0.15^{**}
		(−2.53)
$\text{MPS\_ORTH}\times\text{MPU}_{t-1}$		3.43^{**}
		(1.98)
R^2	0.29	0.31
n_{obs}	290	290

Notes: This table reports estimates from the following regression:

$$R_{t,t+\Delta} = \beta_0 + \beta_I\,\text{MPS\_ORTH}_t + \beta_S\,\text{MPU}_{t-1} + \beta_{\text{Inter}}\,(\text{MPS\_ORTH}_t \times \text{MPU}_{t-1}) + \epsilon_t,$$

where the dependent variable $R_{t,t+\Delta}$ is 100 times the log change in S&P 500 futures over the 30-minute window around FOMC announcements. The dataset spans from February 8, 1990 to December 13, 2023.[a] MPS_ORTH is the orthogonalized monetary policy shock from Bauer and Swanson (2023b) measured during the same window. MPU_{t-1} is monetary policy uncertainty measured one day before FOMC meetings from Eurodollar (later SOFR) options on the 3-month interest rate, interpolated to a 24-month maturity, from Bauer, Lakdawala, and Mueller (2022). Column (1) reports the baseline regression without uncertainty. Column (2) includes MPU and its interaction with MPS_ORTH. Heteroskedasticity and autocorrelation-robust t-statistics are reported in parentheses below coefficient estimates. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

[a]We combine high-frequency S&P 500 data from Marek Jarocinski's website and MPS_ORTH time series on FOMC days from Michael Bauer's website to obtain the longest possible sample for the benchmark regression, as the latter website does not have S&P 500 data after 2019.

As shown in Figure 9, inflation responds negatively to nominal interest rate increases when monetary policy uncertainty is low. However, when monetary policy uncertainty is high inflation and inflation expectations can respond positively to surprise nominal rate increases, especially over longer maturities. Using inflation swap returns on FOMC announcement days as a measure of changes in inflation expectations, we use the following regression to test the Fed information effect on inflation expectations implied by our model:

$$\pi^m_{t,t+\Delta} = \beta_0 + \beta_I\,\Delta i_{t,t+\Delta} + \beta_S\,\text{MPU}_{t-1} + \beta_{\text{Inter}}\big(\Delta i_{t,t+\Delta} \times \text{MPU}_{t-1}\big) + \epsilon_{t,t+\Delta}. \tag{14}$$

In the above Equation, $\pi^m_{t,t+\Delta}$ is the changes in inflation expectation with maturity m measured by inflation swap returns of corresponding maturity on FOMC announcement days. Under the null hypothesis implied by the model, surprise nominal rate increases lower inflation and inflation expectations and as a result, we expect $\beta_I < 0$. In addition, when MPU is high, inflation expectations respond positively to unexpected nominal rate changes, and our models implies $\beta_{Inter} > 0$. Because Fed information effect in our model affect the long-run steady state of the economy, our model implies that the point estimate for β_{Inter} to be significant over longer horizons. The regression results we report in Table 3 are consistent with the above model implications.

Table 3: Inflation Expectations Response to Monetary Policy Shocks and Uncertainty

Maturity	β_I	β_{Inter}	R^2	n_{obs}
year 1	-0.35	0.08	0.03	154
year 5	-0.35^{***}	-0.01	0.11	154
year 7	-0.24^{**}	0.32^{*}	0.03	154
year 8	-0.23^{**}	0.43^{**}	0.04	154
year 9	0.30^{**}	0.28	0.04	154
year 10	-0.28^{**}	0.35^{**}	0.06	154
year 12	-0.33^{**}	0.28^{*}	0.07	154

Notes: This table reports estimates from the following regression:

$$\Delta\pi^m_{t,t+\Delta} = \beta_0 + \beta_I\,\text{MPS\_ORTH}_t + \beta_S\,\text{MPU}_{t-1} + \beta_{\text{Inter}}\big(\text{MPS\_ORTH}_t \times \text{MPU}_{t-1}\big) + \epsilon^m_t,$$

where the dependent variable $\Delta\pi^m_{t,t+\Delta}$ is the daily change in inflation swap rates of maturity m on FOMC announcement days. The dataset spans from August 10, 2004 to November 1, 2023, covering 154 FOMC meetings. MPS_ORTH is the orthogonalized monetary policy shock from Bauer and Swanson (2023b) measured during the 30-minute window around FOMC announcements. MPU_{t-1} is monetary policy uncertainty measured one day before FOMC meetings from Eurodollar (later SOFR) options on the 3-month interest rate, interpolated to a 24-month maturity, from Bauer, Lakdawala, and Mueller (2022). Inflation swap data are from Bloomberg. Each row reports results for a different maturity. Heteroskedasticity and autocorrelation-robust t-statistics are used to compute significance levels: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

Finally, we test our model implication on the Fed information effect on the term structure of interest rates. Similarly to Equation 14, we regress nominal rates of different maturities on measured monetary policy surprises and an interaction term with MPU:

$$R^m_{n,t,t+\Delta} = \beta_0 + \beta_I \, \Delta i_{t,t+\Delta} + \beta_S \, \text{MPU}_{t-1} + \beta_{\text{Inter}} \big(\Delta i_{t,t+\Delta} \times \text{MPU}_{t-1} \big) + \epsilon_{n,t,t+\Delta}. \tag{15}$$

We report the regression results in Table 4.

And we can see from Table 4 that the interaction term is positive, as expected, and the underlying intuition is straightforward–when uncertainty around the long-run interest rate target is high, the public updates more and henceforth the longer the maturity, the changes in nominal yields are more affected by the current level of monetary policy uncertainty.

Table 4: Nominal Yields and Response to Monetary Policy Shocks and Uncertainty

Maturity	β_I	β_{Inter}	R^2	n_{obs}
1 year	0.59***	0.09	0.29	292
5 year	0.51***	0.17	0.16	292
10 year	0.28**	0.21	0.07	292
16 year	0.12*	0.29*	0.04	292
18 year	0.09	0.32*	0.03	292
20 year	0.05	0.35**	0.03	292
25 year	−0.03	0.41**	0.03	292
30 year	−0.10	0.45**	0.02	292

Notes: This table reports estimates from the following regression:

$$\Delta R^m_{n,t,t+\Delta} = \beta_0 + \beta_I \,\text{MPS\_ORTH}_t + \beta_S \,\text{MPU}_{t-1} + \beta_{\text{Inter}} \big(\text{MPS\_ORTH}_t \times \text{MPU}_{t-1}\big) + \epsilon^m_{n,t},$$

where the dependent variable $\Delta R^m_{n,t,t+\Delta}$ is the daily change in nominal Treasury yields of maturity m on FOMC announcement days. The dataset spans from February 8, 1990 to December 13, 2023, covering 292 FOMC meetings. MPS_ORTH is the orthogonalized monetary policy shock from Bauer and Swanson (2023b) measured during the 30-minute window around FOMC announcements. MPU_{t-1} is monetary policy uncertainty measured one day before FOMC meetings from Eurodollar (later SOFR) options on the 3-month interest rate, interpolated to a 24-month maturity, from Bauer, Lakdawala, and Mueller (2022). Yield data are from the Federal website, computed by Gürkaynak, Sack, and Wright (2007). Each row reports results for a different maturity. Heteroskedasticity and autocorrelation-robust t-statistics are used to compute significance levels: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. Table 4 has two more observations than Table 2 because Jarocinski and Karadi (2020) exclude two meetings (January 22 and October 8, 2008) relative to Bauer and Swanson (2023b), as the Fed took significant measures to cope with the Global Financial Crisis during this period. The results in this table are robust to excluding these two meetings.

6 Conclusion

We develop an equilibrium model of monetary policy announcements to account for the Fed information effect—the empirical observation that expansionary monetary policy surprises are sometimes associated with stock market declines and downward revisions of growth forecasts. Our model does not rely on the assumption that the Fed has superior information about economic fundamentals. Instead, we introduce asymmetric information about the Fed's long-run policy objective for the target rate. The model implies that the Fed information effect is more likely to occur when uncertainty about monetary policy is high, and we provide several pieces of empirical evidence supporting this unique implication.

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Appendix

A.1 Additional Figures and Tables

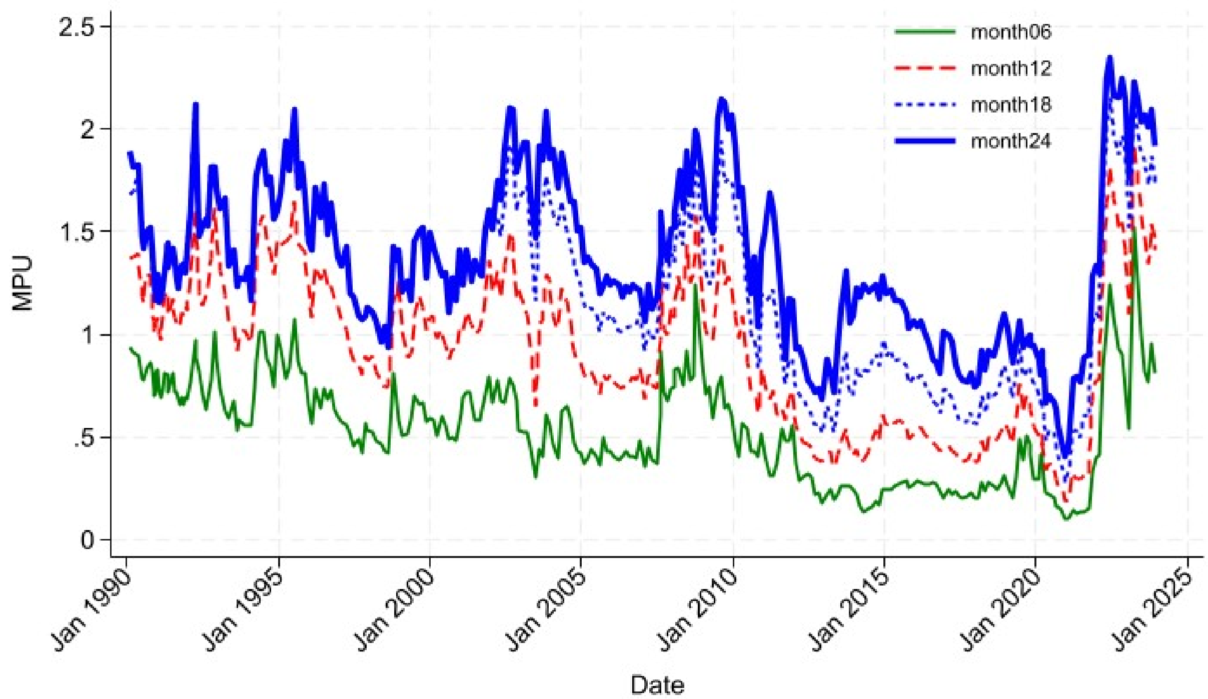

Figure 11: Time Series of Existing Monetary Policy Uncertainty Before FOMC Meetings

Notes: This figure plots the time series of market-based monetary policy uncertainty (MPU) measured one day before each FOMC meeting from January 1990 to December 2023. MPU is calculated as the square root of the model-free variance derived from money market derivatives prices, following the methodology of Bauer, Lakdawala, and Mueller (2022). Four series are shown corresponding to different forecast horizons: 6 months (green solid line), 12 months (red dashed line), 18 months (blue dash-dot line), and 24 months (thick blue solid line). The measure is based on Eurodollar futures and options through September 2022, and SOFR (Secured Overnight Financing Rate) futures and options from October 2022 onwards, reflecting the phasing-out of LIBOR. In our benchmark empirical analysis, we use the 24-month maturity series because the policy uncertainty under study is about long-run short term policy rate uncertainty.

Table 5: Inflation swap responses to monetary policy and information shocks

maturity	$\beta_{\text{mp\_pm}}$	$\beta_{\text{cbi\_pm}}$	$H_0: \beta_{\text{mp\_pm}} - \beta_{\text{cbi\_pm}} = 0$
1 year	−0.32	3.34	−3.66
t-stat	−0.98	1.65	−1.78
2 year	−0.20	2.34	−2.54
t-stat	−1.01	1.39	−1.50
3 year	−0.13	0.44	−1.21
t-stat	−0.79	2.46	−2.59
n_{obs}	163	163	163

Notes: This table reports regression coefficients from estimating the response of inflation swap rate changes to monetary policy shocks (mp_pm) and central bank information shocks (cbi_pm) across different maturities. Rows labeled *t-stat* report t-statistics computed using HAC-robust standard errors with a bandwidth of 4. The third column tests the null hypothesis that the two coefficients are equal. The data span 163 FOMC announcements from 2004 to 2023.[a] Inflation swap data are from Bloomberg, and shock series are from Bauer and Swanson (2023a).

[a]When using decomposed shocks from Jarocinski and Karadi (2020) for inflation swap regressions, we have 7 more observations than in Table 3, which uses shocks from Bauer and Swanson (2023a). This difference occurs because Bauer and Swanson (2023a) exclude all FOMC meetings from March to October 2020 when constructing their shocks, whereas Jarocinski and Karadi (2020) exclude only the unscheduled meetings from this period. The results are robust to excluding these additional meetings.

Table 6: Treasury yield responses to monetary policy and information shocks

maturity	$\beta_{\text{mp\_pm}}$	$\beta_{\text{cbi\_pm}}$	$H_0: \beta_{\text{mp\_pm}} - \beta_{\text{cbi\_pm}} = 0$
2 year	0.62	0.89	−0.27
t-stat	6.63	11.23	−2.25
5 year	0.52	0.78	−0.26
t-stat	5.46	8.14	−1.94
10 year	0.33	0.51	−0.18
t-stat	4.19	6.67	−1.62
30 year	0.18	0.34	−0.16
t-stat	3.05	4.66	−1.72
n_{obs}	297	297	297

Notes: This table reports regression coefficients from estimating the response of Treasury yield changes to monetary policy shocks (mp_pm) and central bank information shocks (cbi_pm) across different maturities. Rows labeled *t-stat* report t-statistics computed using HAC-robust standard errors with a bandwidth of 4. The third column tests the null hypothesis that the two coefficients are equal. The data span 297 FOMC announcements from 1990 to 2023.[a] High-frequency Treasury yield data and shock series are from Bauer and Swanson (2023b).

[a]When using decomposed shocks from Jarocinski and Karadi (2020) for Treasury yield regressions, we have 7 more observations than in Table 2, which uses shocks from Bauer and Swanson (2023a). This difference occurs because Bauer and Swanson (2023a) exclude all FOMC meetings from March to October 2020 when constructing their shocks, whereas Jarocinski and Karadi (2020) exclude only the unscheduled meetings from this period. The results are robust to excluding these additional meetings.

Table 7: Nominal Yields and Monetary Policy Uncertainty: Robustness with Filipović, Pelger, and Ye (forthcoming) Zero-Coupon Yields

Maturity	β_I	β_{Inter}	R^2	n_{obs}
1 year	0.72^{***}	-0.09	0.37	292
5 year	0.66^{***}	0.10	0.21	292
10 year	0.39^{***}	0.19	0.11	292
16 year	0.15	0.39^{**}	0.06	292
18 year	0.12	0.40^{**}	0.05	292
20 year	0.11	0.38^{**}	0.05	292
25 year	0.08	0.37^{**}	0.04	292
30 year	0.06	0.41^{**}	0.04	292

Notes: This table presents robustness results using an alternative measure of nominal Treasury yields. The regression specification is identical to Table 4:

$$\Delta R^m_{n,t,t+\Delta} = \beta_0 + \beta_I \, \text{MPS\_ORTH}_t + \beta_S \, \text{MPU}_t + \beta_{\text{Inter}} \big(\text{MPS\_ORTH}_t \times \text{MPU}_t\big) + \epsilon^m_{n,t},$$

where the dependent variable $\Delta R^m_{n,t,t+\Delta}$ is now the daily change in zero-coupon nominal Treasury yields of maturity m estimated using the nonparametric method of Filipović, Pelger, and Ye (forthcoming), rather than the yields from the Federal Reserve website, computed by the method of Gürkaynak, Sack, and Wright (2007) used in the main specification. All other variables and the sample period (February 8, 1990 to December 13, 2023, covering 292 FOMC meetings) remain identical to the baseline specification. Heteroskedasticity and autocorrelation-robust t-statistics are used to compute significance levels: *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

A.2 Solution of the Markov Equilibrium

The solution of our model is related to the literature on macroeconomic models with financial frictions, especially continuous-time models such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013). With monopolistic competition, we cannot solve the equilibrium as a social planner's problem and equilibrium prices and quantities have to be simultaneously determined by a set of equilibrium conditions. We borrow the equilibrium selection device of Angeletos and Lian (2023) and focus on Markov equilibrium with minimal state variables.

Combing the production function (11) and the resource constraint (12), we can write consumption as a function of labor supply:

$$C\left(\hat{\theta},\nu\right)=\left[1-h\left(\pi\left(\hat{\theta},\nu\right)\right)\right]AL\left(\hat{\theta},\nu\right). \tag{16}$$

Assuming log preference, i.e. $u\left(C\right)=\ln C$, Equation (10) can be used to write $L\left(\hat{\theta},\nu\right)$ as a function of real wage:

$$\left[1-h\left(\pi\left(\hat{\theta},\nu\right)\right)\right]AL^{1+\zeta}\left(\hat{\theta},\nu\right)=w\left(\hat{\theta},\nu\right). \tag{17}$$

The above two equations together allows us to solve consumption as a function of real wage and write the risk-free rate as:

$$r\left(\hat{\theta},\nu\right)=\beta-\frac{\mathcal{L}\left\{\left[A\left(1-\frac{1}{2}h\left(\pi\left(\hat{\theta},\nu\right)\right)\right)\right]^{-\frac{\zeta}{1+\zeta}}w\left(\hat{\theta},\nu\right)^{-\frac{1}{1+\zeta}}\right\}}{\left[A\left(1-\frac{1}{2}h\left(\pi\left(\hat{\theta},\nu\right)\right)\right)\right]^{-\frac{\zeta}{1+\zeta}}w\left(\hat{\theta},\nu\right)^{-\frac{1}{1+\zeta}}}. \tag{18}$$

Here, the operator $\mathcal{L}$ is defined as $\mathcal{L}X_t=\lim_{\Delta\to 0}\frac{1}{\Delta}E_t\left[X_{t+\Delta}-X_t\right]$ for any random variable X where the limit is defined.

Proposition 1. *(Markov Equilibrium)*

Real wage and real interest rate $\left\{w\left(\hat{\theta},\nu\right),r\left(\hat{\theta},\nu\right)\right\}$ *are jointly determined as the solution to the following two functional equations. The first is the optimal pricing setting* *equation:*

$$\rho h'\left(\pi\left(\hat{\theta},\nu\right)\right)=(\eta-1)\left[\hat{\mu}\frac{w\left(\hat{\theta},\nu\right)}{A}-1\right]+\mathcal{L}\left[h'\left(\pi\left(\hat{\theta},\nu\right)\right)\right], \tag{19}$$

where $\hat{\mu}=\frac{\eta}{\eta-1}$ *is the optimal markup in a flexible price equilibrium. The second equation is the requirement of Taylor rule:*

$$\beta-\frac{\mathcal{L}\left\{\left[A\left(1-\frac{1}{2}h\left(\pi\left(\hat{\theta},\nu\right)\right)\right)\right]^{-\frac{\zeta}{1+\zeta}}w\left(\hat{\theta},\nu\right)^{-\frac{1}{1+\zeta}}\right\}}{\left[A\left(1-\frac{1}{2}h\left(\pi\left(\hat{\theta},\nu\right)\right)\right)\right]^{-\frac{\zeta}{1+\zeta}}w\left(\hat{\theta},\nu\right)^{-\frac{1}{1+\zeta}}}=(\phi-1)\,\pi\left(\hat{\theta},\nu\right)+\nu. \tag{20}$$

Given $\left\{w\left(\hat{\theta},\nu\right),r\left(\hat{\theta},\nu\right)\right\}$, *we can recover labor supply from (17), consumption from (16), risk-free interest rate from (18) and the output function from (11).*

The key mechanism of the Fed information effect of our model operates through Equation (19). As we show in Appendix A, this equation is the optimality condition for firms' pricing setting decisions. The left hand side of (19) represents the marginal cost of price adjustment. The term $(\eta-1)\left[\hat{\mu}\frac{w\left(\hat{\theta},\nu\right)}{A}-1\right]$ is the current period flow benefit of price adjustment. Note that $\hat{\mu}$ is optimal markup under the flexible price equilibrium. $\hat{\mu}\frac{w\left(\hat{\theta},\nu\right)}{A}>1$ implies that markup is lower than the flexible price optimum. In this case, the marginal benefit of raising prices is positive. For the same reason, $\hat{\mu}\frac{w\left(\hat{\theta},\nu\right)}{A}<1$ corresponds to the case where markup is lower than the flexible price optimum, and the marginal benefit of raising price is negative. The term $\mathcal{L}\left[h'\left(\pi\left(z\right)\right)\right]$ represents expected future benefit of price adjustment.

By equation (19), keeping the expectation about future inflation, $\mathcal{L}\left[h'\left(\pi\left(\hat{\theta},\nu\right)\right)\right]$, fixed, a rise in inflation $\pi\left(\hat{\theta},\nu\right)$ must be associated with a rise in real wage $w\left(\hat{\theta},\nu\right)$ and therefore employment and output. This is the standard expansionary effect of monetary policy in New Keynesian models. Consider now the effect of an information about $\hat{\theta}$ keeping current inflation fixed. A expectation of a higher $\hat{\theta}$ is associated with a higher long-run interest rate target and a lower inflation expectation. Keeping current inflation constant, a lower inflation expectation must imply a lower $\mathcal{L}\left[h'\left(\pi\left(\hat{\theta},\nu\right)\right)\right]$ and a higher real wage $w\left(\hat{\theta},\nu\right)$. In this case, a rise in interest rate can be associated with a higher real wage, a greater employment, and an increase in output.

A.3 Derivation of Asset Prices from the Model

A.3.1 Equity Prices

To compute asset prices, we first compute the pricing kernel:

$$\Lambda_t = e^{-\rho t} C\left(\hat{\theta}_t, \nu_t\right)^{-\gamma},$$

where $C\left(\hat{\theta}_t, \nu_t\right) = \left[1 - h\left(\hat{\theta}_t, \nu_t\right)\right] A^{\frac{\zeta}{1+\zeta}} w\left(\hat{\theta}_t, \nu_t\right)^{\frac{1}{1+\zeta}}$ is the consumption policy.

Note that firm value, $V\left(\hat{\theta}, \nu\right)$ is given by (4). In equilibrium, $p\left(\hat{\theta}, \nu\right) = P\left(\hat{\theta}, \nu\right)$, and this equation can be written recursively and greatly simplified:

$$\rho V\left(\hat{\theta}, \nu\right) = \Psi\left(1 | \, w\left(\hat{\theta}, \nu\right), 1\right) - h\left(\pi\left(\hat{\theta}, \nu\right)\right) + \mathcal{L} V\left(\hat{\theta}, \nu\right),$$

where

$$\Psi\left(1 | \, w\left(\hat{\theta}, \nu\right), 1\right) = \left(\frac{p}{P}\right)^{-\eta} Y \left[\frac{p}{P} - \frac{\bar{k}(w)}{A}\right]\Bigg|_{p=P;Y=1} = 1 - \frac{w\left(\hat{\theta}, \nu\right)}{A}.$$

is the per period profit of the firm. There are two simplifications here. First, $p_t = P_t$ so that all firms are identical. Second, under the assumption of log preference, we can work with the normalized value function and the normalized profit function. Once we computed $V\left(\hat{\theta}, \nu\right)$, the firm's equity value is given by: $\bar{V}\left(\hat{\theta}, \nu, Y\right) = V\left(\hat{\theta}, \nu\right) Y\left(\hat{\theta}, \nu\right)$. In the learning case with regime switch, we have two continuous state variables, and the firm value function equation becomes:

$$\rho V\left(\hat{\theta}, \nu\right) = 1 - \frac{w\left(\hat{\theta}, \nu\right)}{A} - \frac{h_0}{2}\pi^2\left(\hat{\theta}, \nu\right) + \mathcal{L} V\left(\hat{\theta}, \nu\right)$$

A.3.2 Expected Inflation

Let $\pi\left(\hat{\theta}, \nu\right)$ be the policy function of inflation. Fix a τ, for any $t < \tau$, define:

$$\tilde{\pi}\left(\hat{\theta}, \nu, t; \tau\right) = E\left[\pi\left(\hat{\theta}_\tau, \nu_\tau\right) \Big| \, \hat{\theta}_t = \hat{\theta}, \nu_t = \nu\right].$$

Here, $\left\{\tilde{\pi}\left(\hat{\theta}, \nu, t; \tau\right)\right\}_{t\in(0,\tau)}$ is a family of conditional expectations. Clearly, $\tilde{\pi}\left(\hat{\theta}_t, \nu_t, t; \tau\right)$ is a martingale. This function can be determined by:

1. Boundary condition: $\tilde{\pi}\left(\hat{\theta}, \nu, \tau; \tau\right) = \pi\left(\hat{\theta}, \nu\right)$ for all $\left(\hat{\theta}, \nu\right)$
2. $\mathcal{L}\tilde{\pi}\left(\hat{\theta}_t, \nu_t, t; \tau\right) = 0$ in the interior of $(nT, (n+1)\,T)$
3. At nT: $\tilde{\pi}\left(\hat{\theta}, \nu, nT; \tau\right) = E\left[\tilde{\pi}\left(\hat{\theta}_{nT}^{+}, \nu_{nT}^{+}, nT; \tau\right)\middle|\, \hat{\theta}_{nT} = \hat{\theta}, \nu_{nT} = \nu\right]$

Once we computed $\tilde{\pi}\left(\hat{\theta}, \nu, t; \tau\right)$, we plot:

$$\tilde{\pi}\left(\hat{\theta}_0^{+}, \nu_0^{+}, 0^{+}; \tau\right) - \tilde{\pi}\left(\hat{\theta}_0, \nu_0, 0; \tau\right),$$

for a set of maturities, τ.

PDE for Expected Inflation:

Since $\tilde{\pi}\left(\hat{\theta}_t, \nu_t, t; \tau\right)$ is a martingale when we fix $\tau = 1, 2, \ldots$, we have:

$$\begin{aligned}
0 = {} & \frac{\partial\tilde{\pi}(\hat{\theta}_t, \nu_t, t; \tau)}{\partial t} + a(\hat{\theta}_t - \nu_t)\,\tilde{\pi}_{\nu}(\hat{\theta}_t, \nu_t, t; \tau) \\
& + (\lambda_H + \lambda_L)(\bar{\theta} - \hat{\theta}_t)\,\tilde{\pi}_{\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau) \\
& + a(\theta_H - \hat{\theta}_t)(\hat{\theta}_t - \theta_L)\,\tilde{\pi}_{\nu\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau) \\
& + \tfrac{1}{2}\sigma_{\nu}^2\,\tilde{\pi}_{\nu\nu}(\hat{\theta}_t, \nu_t, t; \tau) \\
& + \tfrac{1}{2}\frac{a^2}{\sigma_{\nu}^2}(\theta_H - \hat{\theta}_t)^2(\hat{\theta}_t - \theta_L)^2\,\tilde{\pi}_{\hat{\theta}\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau).
\end{aligned}$$

For the inflation expectation, we compute backwards just as for the yields. We write:

$$\frac{\partial\tilde{\pi}\left(\hat{\theta}_t, \nu_t, t; \tau\right)}{\partial t} = \frac{\tilde{\pi}_t - \tilde{\pi}_{t-\Delta t}}{\Delta t}.$$

Denote the drift terms as:

$$\begin{aligned}\widehat{\pi}_t^{\text{Terms}} &= a(\hat{\theta}_t - \nu_t)\,\tilde{\pi}_\nu(\hat{\theta}_t, \nu_t, t; \tau) + (\lambda_H + \lambda_L)(\bar{\theta} - \hat{\theta}_t)\,\tilde{\pi}_{\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau) \\ &+ a(\theta_H - \hat{\theta}_t)(\hat{\theta}_t - \theta_L)\,\tilde{\pi}_{\nu\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau) \\ &+ \tfrac{1}{2}\sigma_\nu^2\,\tilde{\pi}_{\nu\nu}(\hat{\theta}_t, \nu_t, t; \tau) \\ &+ \tfrac{1}{2}\frac{a^2}{\sigma_\nu^2}(\theta_H - \hat{\theta}_t)^2(\hat{\theta}_t - \theta_L)^2\,\tilde{\pi}_{\hat{\theta}\hat{\theta}}(\hat{\theta}_t, \nu_t, t; \tau).\end{aligned}$$

Then start from $\tilde{\pi}_\tau = \pi\left(\hat{\theta}_\tau, \nu_\tau\right)$ and recursively solve for $\tilde{\pi}_{t-\Delta t}$ as $\tilde{\pi}_t + \Delta t \times \widehat{\pi}_t^{\text{Terms}}$, working backwards up to 20 years earlier.

A.3.3 Nominal Yields

Consider a one-dollar payment at time T. The price of a nominal bond is equivalent to the present value of this payoff at time t is given by:

$$Q\left(t, \hat{\theta}_t, \nu_t\right) = E_t\left[\frac{\Lambda_T}{\Lambda_t}\frac{P_t}{P_T}Q\left(T, \hat{\theta}_T, \nu_T\right)\right] = E_t\left[e^{-\rho(T-t)}\left[\frac{C\left(\hat{\theta}_T, \nu_T\right)}{C\left(\hat{\theta}_t, \nu_t\right)}\right]^{-\gamma} e^{-\int_t^T \pi\left(\hat{\theta}_s, \nu_s\right)ds}\right].$$

This implies that:

$$\frac{\Lambda_t Q\left(t, \hat{\theta}_t, \nu_t\right)}{P\left(t, \hat{\theta}_t, \nu_t\right)} = E_t\left[\frac{\Lambda_T}{P_T}\right].$$

Clearly, $\frac{\Lambda_t Q\left(t, \hat{\theta}_t, \nu_t\right)}{P\left(t, \hat{\theta}_t, \nu_t\right)}$ is a martingale, and we have:

$$\mathcal{L}\left[\frac{\Lambda_t Q\left(t, \hat{\theta}_t, \nu_t\right)}{P\left(t, \hat{\theta}_t, \nu_t\right)}\right] = 0. \tag{21}$$

This yields:

$$\frac{\partial Q\left(t, \hat{\theta}, \nu\right)}{\partial t} + Q^{\text{terms}} = \frac{Q\left(t, \hat{\theta}, \nu\right) - Q\left(t - \Delta t, \hat{\theta}, \nu\right)}{\Delta t} + Q^{\text{terms}} = 0,$$

and we can get:

$$Q\left(t-\Delta t,\hat{\theta},\nu\right)=\Delta t\times Q^{\text{terms}}+Q\left(t,\hat{\theta},\nu\right).$$

where Q^{terms} is similar to $\widehat{\pi}_t^{\text{Terms}}$ from the computation of inflation expectation. We compute backwards all the way to 20 years before T, and this corresponds to the price of a 20-year-maturity zero-coupon bond. Given $\hat{\theta}$ and ν, we can compute the term structure $y\left(\hat{\theta},\nu;t\right)$ as a function of maturity t.